B.Tech.
(SEM. I) ODD SEMESTER THEORY EXAMINATION 2012-13
MATHEMATICS—I

Time : 3 Hours  Total Marks : 100

SECTION—A

1. All parts for this question are compulsory:  
   \((2\times10=20)\)
   (a) Find the 8th derivative of \(x^2e^x\).
   (b) If \(x^2 = au + bv, y^2 = au - bv\), then find \(\left(\frac{\partial u}{\partial x}\right)_y \left(\frac{\partial x}{\partial u}\right)_v\).
   (c) Find the stationary points of
   \[f(x, y) = 5x^2 + 10y^2 + 12xy - 4x - 6y + 1.\]
   (d) If \(x = u(1 + v), y = v(1 + u)\), then find the Jacobian of \(u, v\) with respect to \(x, y\).
   (e) Reduce the matrix
   \[
   \begin{bmatrix}
   1 & 1 & 1 \\
   3 & 1 & 1 
   \end{bmatrix}
   \]
   into normal form.
   (f) Prove that the matrix
   \[A = \frac{1}{\sqrt{3}} \begin{bmatrix}
   1 & 1 + i \\
   1 - i & -1 
   \end{bmatrix}\]
   is unitary.
   (g) Evaluate
   \[\int_0^1 \int_0^1 xe^{xy} \, dx \, dy.\]
   (h) Evaluate \(\Gamma(-3/2)\).
   (i) Find the value of \(m\) if \(\vec{F} = mxi - 5yj + 2zk\) is a solenoidal vector.
(j) Find the unit normal at the surface \( z = x^2 + y^2 \) at the point (1, 2, 5).

SECTION—B

2. Attempt any three parts of the following: \((3 \times 10 = 30)\)

(a) If \( y = \left( x + \sqrt{1 + x^2} \right)^m \), then find the \( n \)th derivative of \( y \) at \( x = 0 \).

(b) Find the maximum and minimum distance of the point (1, 2, -1) from the sphere \( x^2 + y^2 + z^2 = 24 \).

(c) Find the eigen values and eigen vectors of the following matrix:

\[
\begin{bmatrix}
3 & 10 & 5 \\
-2 & -3 & -4 \\
3 & 5 & 7
\end{bmatrix}
\]

(d) Evaluate \( \iiint_V (ax^2 + by^2 + cz^2) \, dx \, dy \, dz \) where \( V \) is the region bounded by \( x^2 + y^2 + z^2 \leq 1 \).

(e) Verify Gauss’s divergence theorem for the function \( \vec{F} = x^2 \hat{i} + \hat{j} + yz \hat{k} \) over unit cube.

SECTION—C

Attempt any two parts from each question of this section. All questions are compulsory. \( [(2 \times 5) \times 5 = 50] \)

3. (a) State and prove Euler’s theorem for homogeneous functions.

(b) Expand \( f(x, y) = e^x \tan^{-1} y \) in powers of \((x - 1)\) and \((y - 1)\) up to two terms of degree 2.
(c) If \( z = f(x, y) \) where \( x = e^u \cos v, y = e^u \sin v \), prove that
\[
\left( \frac{\partial f}{\partial x} \right)^2 + \left( \frac{\partial f}{\partial y} \right)^2 = e^{-2u} \left[ \left( \frac{\partial f}{\partial u} \right)^2 + \left( \frac{\partial f}{\partial v} \right)^2 \right].
\]

4. (a) If \( x + y + z = u, y + z = uv, z = uvw \), then find \( \frac{\partial (x, y, z)}{\partial (u, v, w)} \).

(b) The two sides of a triangle are measured as 50 cm and 70 cm, and the angle between them is 30\(^\circ\). If there are possible errors of 0.5\% in the measurement of the sides and 0.5 degree in that of the angle, find the maximum approximate percentage error in measuring the area of the triangle.

(c) Show that \( u = y + z, v = x + 2z^2, w = x - 4yz - 2y^2 \) are not independent. Find the relation between them.

5. (a) Test the consistency and hence, solve the following set of equations:
\[
\begin{align*}
10y + 3z &= 0, \\
3x + 3y + 2z &= 1, \\
2x - 3y - z &= 5, \\
x + 2y &= 4.
\end{align*}
\]

(b) Using elementary transformations, find the rank of the following matrix:
\[
A = \begin{bmatrix}
-2 & -1 & 3 & -1 \\
1 & 2 & -3 & -1 \\
1 & 0 & 1 & 1 \\
0 & 1 & 1 & -1
\end{bmatrix}
\]

(c) Examine the following vectors for linearly dependent and find the relation between them, if possible:
\( X_1 = (1, 1, -1, 1), X_2 = (1, -1, 2, -1), X_3 = (3, 1, 0, 1) \).
(j) Find the unit normal at the surface \( z = x^2 + y^2 \) at the point 
(1, 2, 5).

**SECTION—B**

2. Attempt any three parts of the following: \((3 \times 10 = 30)\)

(a) If \( y = \left( x + \sqrt{1 + x^2} \right)^m \), then find the \( n \)th derivative of \( y \) at 
\( x = 0 \).

(b) Find the maximum and minimum distance of the point 
(1, 2, -1) from the sphere \( x^2 + y^2 + z^2 = 24 \).

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(d) Evaluate \( \iiint_V (ax^2 + by^2 + cz^2) \, dx \, dy \, dz \) where \( V \) is the 
region bounded by \( x^2 + y^2 + z^2 \leq 1 \).

(e) Verify Gauss's divergence theorem for the function 
\( \mathbf{F} = x^2 \mathbf{i} + y \mathbf{j} + z \mathbf{k} \) over unit cube.

**SECTION—C**

Attempt any two parts from each question of this section. All questions are compulsory. \([2 \times 5 \times 5 = 50]\)

3. (a) State and prove Euler's theorem for homogeneous functions.

(b) Expand \( f(x, y) = e^x \tan^{-1} y \) in powers of \( (x - 1) \) and \( (y - 1) \) 
upto two terms of degree 2.

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